Bayesian estimation of two-dimensional psychophysical functions

Theoretical framework

Psychometric function

\[ p_r = \Psi(y, x, S) \]

Psychophysical/critical function

\[ S = x \cdot \gamma + \delta \]

Likelihood

\[ P(r | y, x, S) = \frac{p_r}{1 - p_r} \]

Posterior

\[ E_{\alpha, \beta}(x, y) = P(x, y | S) \]

Estimating a 2D response probability surface (e.g., contrast sensitivity function)

\[ y \sim \Psi(x, y) \]

Assume that slope of the psychometric function is invariant to translation.

\[ P(x, y) = \Psi(y) \pm \delta \]

5 pspsychometric slope parameter.

\[ \gamma \sim \text{critical value, translation parameter for psychometric function} \]

\[ \alpha \sim \text{psychophysical critical function that determines sensitivity} \]

\[ \beta \sim \text{psychophysical critical function parameters.} \]

Goal of Experiments: estimate \( \theta \) (and perhaps \( \phi \))

Simulations

2AFC Cumulative Normal psychometric function

Single value criterion (critical function)

\[ p_r = \frac{1}{1 + e^{-\left( x - \mu \right)/\sigma}} \]

Mean squared error: regardless of systematic bias, how far off from the true answer are we?

\[ \text{Error} = \left( x - \mu \right)^2 \]

logistic psychometric function

\[ p_r = \frac{1}{1 + e^{-\left( x - \mu \right)/\sigma}} \]

Exponential decay critical function

\[ p_r = e^{-\left( x - \mu \right)/\sigma} \]

Infinities in first 200 trials slow convergence, but, of course, all sensible methods of trial selection should converge to the same estimates.

Does it work? Single threshold (Psi; Kontsevich & Tyler)

2AFC Weibull psychometric function

Single value criterion (critical function)

\[ p_r = \frac{1}{1 + e^{-\left( x - \mu \right)/\sigma}} \]

\[ \text{Error} = \left( x - \mu \right)^2 \]

Does it work? TvC function (qTvC; Lesmes et al)

2D threshold estimation

Traditional Methods:

- Constant stimuli
- Limits
- Adjustment (inefficient)

Newer Methods:

- Staircases of various forms
- QUEST, PSI, etc.

(inefficient on account of avoiding stimulus strengths where \( p(\text{response}) = 0 \) or 1)

Traditional Methods:

- 1000 (or so) staircases

Newer Methods:

- Ellipse estimation
- \( qTVc \)

- 1000 (or so) staircases

Feedback.

FAST:

- Ellipse estimation
- \( qTVc \)

Matlab Toolbox

Easy to use implementation; for example:

Defining a structure:

\[ \text{test} = \text{FastStruct}('\text{testname}', 0, \text{plogistic}, \{ 0.5 \times 0.5, \{ \text{0.01, 0.1}, \{ \text{1.0, 1.0}, \{ \text{1.0, 1.0} \}} \}) \]

Choosing a stimulus (local entropy minimization):

\[ \text{y} = \text{FastChooseYent}('\text{test}, 3, \{ \text{-10, 10} \}) \]

Updating the likelihoods:

\[ \text{test} = \text{FastUpdate}('\text{test}, 5 \times 5 \times 5) \]

Resampling parameter lattice:

\[ \text{test} = \text{FastSample}\text{struct}('\text{test}) \]

Evaluating performance

Bias: Is the estimation procedure causing systematic deviations in one direction or another?

\[ \Delta x = \frac{1}{N} \sum_{i=1}^{N} \left( x_i - \mu \right) \]

Why is this more efficient than 1000 staircases?

Logistic psychometric function

\[ p_r = \frac{1}{1 + e^{-\left( x - \mu \right)/\sigma}} \]

Exponential decay critical function

\[ p_r = \exp\left( -\frac{x - \mu}{\sigma} \right) \]

Global minimizing Posterior Entropy:

\[ \text{Ent} = \sum_{i=1}^{N} \left( p(x_i | \theta) \log p(x_i | \theta) \right) \]

Ironically, less informative?

Results

Contrast Sensitivity Function

\[ \text{Mean squared error: regardless of systematic bias, how far off from the true answer are we?} \]

\[ \text{Error} = \left( x - \mu \right)^2 \]

\[ \mu = \text{threshold} \]

\[ \sigma = \text{variance} \]

\[ \text{Mean marginals and quadratic approximation confidence interval calibration.} \]

\[ \text{Globally minimizing Posterior Entropy} \]

Vernier Acuity (as a function of eccentricity)

\[ \text{Hyperbolic discounting rate.} \]

\[ \text{Exponential Timecourse (McCollough effect)} \]

Contact:

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Background and Theory

Functional Adaptive Sequential Testing: Bayesian estimation of two-dimensional psychophysical functions

1D threshold estimation

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- Limits
- Adjustment (inefficient)

Newer Methods:

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2D threshold estimation

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Implementation

Kuk for details, motivation

Updating posterior

Grid search (not MCMC or Particle Filter)

Selecting a stimulus

1. Minimize expected posterior entropy

2. Minimize expected variance of posterior

3. Present at a quantile.

Evaluating parameters

1. Marginal means

2. Marginal Maximum A Posteriori (MAP)

3. Lattice MAP

Making grid search better

Dynamically resample parameter lattice.